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# LETTERS TO THE EDITOR

## DISCUSSION ON "STOCHASTIC STABILITY OF QUASI-NON-INTEGRABLE-HAMILTONIAN SYSTEMS"

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This letter is intended as a discussion on the above paper [1]. There are several points this writer wishes to raise and hopefully they will be clarified.

Firstly, in the deterministic context, it is well known that for holonomic non-conservative systems, the Hamiltonian H is not constant implying that the motion integral in the form of the Jacobi integral no longer exists [2], while the essence of averaging is hinged on the existence of an action integral. Clearly, for multi-degrees-of-freedom (mdof) holonomic non-conservative nonlinear systems under parametric random excitations this issue is more complex. Does the analysis in section 3 of the above paper include this class of problems?

Secondly, this writer does not agree with the authors that the Hamiltonian of the system in section 5 of reference [1] is

$$H = \tfrac{1}{2}(\dot{X}^2 + \dot{Y}^2) + U(X, Y), \tag{1}$$

where the symbols are defined in reference [1] and shall not be repeated here.

The right hand side (rhs) of the above equation should have a term contributed by the parametric random excitations associated with the restoring forcing terms of the system considered. By disregarding the contribution of the parametric random excitations, the dynamics of the resulting two degrees-of-freedom (dof) nonlinear system is entirely different. Furthermore, "for quasi-nonintegrable-Hamiltonian system with parametric excitations of Gaussian white noises, these boundaries are often singular" (the quote here is from the last sentence of the incomplete paragraph below equation (28) of reference [1]). Clearly, by disregarding the parametric random excitations, even the boundaries will be very much different from those of the original nonlinear system.

Thirdly, the averaged Hamiltonian is governed by the Itô equation (17) of reference [1] in order to perform the classification of singular boundaries (c.s.b.) for the $Y = H^{1/2}$ process alone. It is not clear to this writer as to what restriction(s) there is(are) for the associated solutions of the Cauchy's problems. Put it differently, what proper structures of the nonlinearities and terms associated with the parametric random excitations of the mdof nonlinear systems can lead to equation (17)? This leads to the following related question.

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Finally, the boundedness of solutions for $X$ and $Y$ in the governing equation of motion of the two dof nonlinear system considered in section 5 of reference [1], is crucial in arriving at equation (17) of reference [1]. The question, then, is: can one guarantee the boundedness of the responses of mdof nonlinear systems even if one can disregard the parametric random excitations? If the answer is affirmative, the next logical question is: how?

## REFERENCES

1. W. Q. Zhu and Z. L. Huang 1998 *Journal of Sound and Vibration* **218**, 769–789. Stochastic stability of quasi-non-integrable-Hamiltonian systems.
2. L. Meirovitch 1970 *Methods of Analytical Dynamics*, p. 247. New York: McGraw-Hill.

# AUTHORS' REPLY

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The present authors thank Professor To for his interest in our paper [1]. We are delighted to discuss the points raised by him.

In our paper, a quasi-Hamiltonian system is formulated as a Hamiltonian system subject to light dampings and weak stochastic excitations. The Hamiltonian systems considered are holonomic and conservative, and the Hamiltonian $H = H(\mathbf{q}, \mathbf{p})$ is independent of time. Non-conservative Hamiltonian systems are not considered.

For the example in section 5 of our paper [1], the Hamiltonian is

$$H = \tfrac{1}{2}(\dot{X}^2 + \dot{Y}^2) + U(X, Y), \tag{1}$$

since the parametric excitations are treated as the perturbation to the Hamiltonian system rather than as a part of the Hamiltonian system, and the Wong–Zakai correction terms vanish. For an averaged Hamiltonian, the boundaries are often singular just because the Hamiltonian system is subjected to parametric excitations of Gaussian white noises. Here, the effect of parametric excitations on the behavior of boundaries has been considered. Again, the parametric excitations are treated as the perturbation to the Hamiltonian system rather than a part of the Hamiltonian system.

The averaged equation (17) in our paper is derived for quasi-non-integrable-Hamiltonian systems based on a theorem due to Khasminskii [2]. The conditions are that the Hamiltonian systems are conservative and non-integrable, the dampings are light and the stochastic excitations are weak. The boundedness of solutions for $X$ and $Y$ is not a necessary condition for deriving the averaged